Introduction to Integration

Math 130 - Essentials of Calculus

16 April 2021

The Fundamental Theorem of Calculus

Let's watch a video!

https://youtu.be/rfG8ce4nNh0

We will discuss some questions afterward.

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7. What is the Fundamental Theorem of Calculus?

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6. How are antiderivatives useful in this problem?
7. What is the Fundamental Theorem of Calculus?
8. What happens if the function we integrate takes on negative values? What does the integral compute here?

Integral Terminology

In the integral

$$\int_a^b f(x)\,dx$$

- $f(x)$ is called the **integrand**
- a and b are called the **limits of integration**
- specifically, a is the **lower limit** and b is the **upper limit**

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Using integration, we are even able to see how production costs would increase if we wanted to increase the amount of product produced. For example, if we wanted to see the additional costs involved in raising production from 200 to 400 units, we would just compute

$$\int_{200}^{400} M(q)\,dq.$$

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6. $\displaystyle\int_{-1}^0 (2x - e^x)dx$